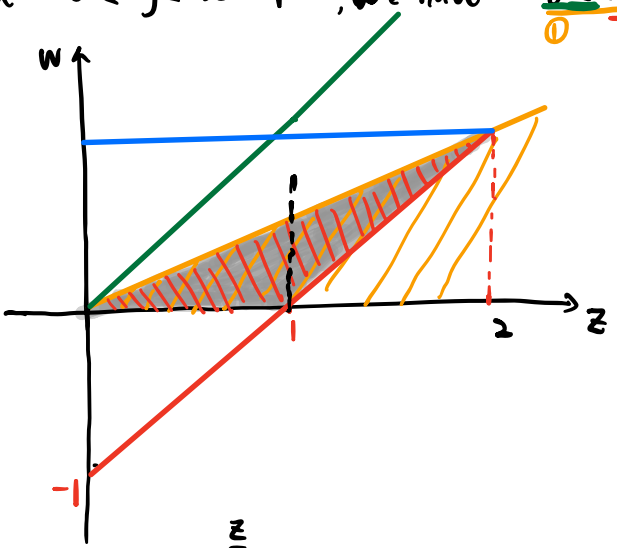
1. (b) $Z = X + Y$

Auxiliary variable $W = Y$ $\Rightarrow$ $\begin{cases} x = z - w \\ y = w \end{cases}$

$$f_{Z,W}(z,w) = f_{X,Y}(x,y)\frac{1}{J(x,y)} = f_{X,Y}(z-w,w)\frac{1}{J(x,y)}$$

Given $0 < y < x < 1$, we have $0 < w < z-w < 1$ (④; ① $0<w$, ② $z-w<1$)




① $0 < w < z-w \Rightarrow 0 < w < \frac{z}{2}$

② $w < z-w < 1 \Rightarrow z-1 < w < \frac{z}{2}$

③ $0 < z-w < 1 \Rightarrow z-1 < w < z$

④ $0 < w < 1$

$$f_Z(z) = \begin{cases} \int_0^{\frac{z}{2}} f_{Z,W}(z,w)\,dw, & 0 < z < 1 \\ \int_{z-1}^{\frac{z}{2}} f_{Z,W}(z,w)\,dw, & 1 \le z < 2 \\ 0, & \text{else} \end{cases}$$